Medical Journal of the
Islamic Republic of Iran

Volume 16
Number 4
Winter 1381
February 2003

# THEORETICAL MODELS FOR DETERMINING THE VELOCITY OF ULTRASOUND IN BONE AND BONE SAMPLES

**M.B. TAVAKOLI**

*From the Department of Medical Physics and Medical Engineering, School of Medicine, Isfahan University of Medical Sciences, Isfahan, I.R. Iran.*

## ABSTRACT

Currently available non-invasive clinical methods for diagnosing osteoporosis are mainly associated with ionizing radiation. Ultrasonic assessment of bone disease is a more recent technology. A theoretical explanation for the relationship between ultrasonic parameters and bone structure is necessary.

Thus, two types of bone samples of perspex (HAP) and perspex-glass beads were produced. Each type was made with a different fraction of constituents. The ultrasonic properties of these samples were measured.

The results of velocity measurement showed rise of velocity in the samples when the volume fraction of the HPA or glass beads increased with a different trend.

A theoretical model for the velocity in a composite material was developed and the results of the two samples were used to test the model. The results showed a good agreement for the perspex-glass bead sample at different volume fraction, while the agreement between perspex-HAP was not as good as the other sample.
*MJIRI, Vol. 16, No. 4, 221-225, 2003.*

**Keywords:** ultrasound, velocity, attenuation, bone sample.

## INTRODUCTION

A primary clinical problem today concerns the early detection of osteoporosis. Currently available non-invasive clinical methods for detection of osteoporosis include: SPA(single photon absorptiometry) of cortical bone, DPA(dual photon absorptiometry) of the spine, and QCT(quantitative computerized tomography). All of these methods are associated with ionising radiation.[1,2]

Ultrasonic assessment of bone disease is a more recent technology. Two ultrasonic parameters involved, attenuation and velocity, have been measured in normal and pathological bone in both cortical and cancellous bone by a number of researchers.[3,6]

The previous measurements have shown different specificity and sensitivity of these parameters due to the bone density and structure. Although several workers[7,8] attempted to solve this problem, there is no theoretical explanation for the mechanism of ultrasonic attenuation and velocity in bone as it is extremely complicated. Perhaps one approach is to measure attenuation and velocity in a synthetic bone phantom under controlled parameters, and monitoring each effect of the bone parameters on attenuation and velocity,

**Table I:** The quantities of physical parameters of bone components and materials used for the bone samples.[9]

| | $\rho$(g cm$^{-3}$) | v(ms$^{-1}$) | Z(kgm$^{-2}$ s$^{-1}$) | $\alpha$(dBcm$^{-1}$ MHZ$^{-1}$) |
|---|---|---|---|---|
| Fat | 0.94 | 1450 | $1.36\times 10^6$ | 0.6 |
| Collagen | 1.43 | 2100 | $3\times10^6$ | 1 |
| HAP | 3.17 | 6400 | $19\times10^6$ | 2 |
| Perspex | 1.2 | 2700 | $3.2\times10$ | 2 |
| Glass(soda) | 2.5 | 5600 | $14\times10^6$ | 0.5< |



[ Downloaded from mjiri.iums.ac.ir on 2026-11-22 ]

In the following section, a simple theoretical model is described to explain the velocity of ultrasound in a composite material as a function of velocity of ultrasound in its constituents and the fraction of each component.

Consider a layer of composite with two constituent materials with thickness of d (Fig. 2). One of the constituents is taken to be the background matrix and the other to be the embedded material. In the case of perspex-HAP or perspex - glass beads the matrix is assumed to be the perspex while in bone, fat is playing a similar role.

Let a plane ultrasonic wave generated by a transducer (transmitter) located on one side propagate through the sample and be detected by another transducer (receiver) located at the opposite side of the sample.

The delay between transmission and arrival of the leading edge of signal at the second transducer will be close to the minimal necessary time for a wave to pass through the sample, say path A in Fig. 2. The transit time of the signal through the sample depends on the amount of the matrix and the embedded material in the matrix traversed by the ultrasound signal. Therefore, if the velocity of ultrasound in the embedded material is greater than in the matrix, the more embedded particles there are present, the earlier the ultrasound signal arrives at the second transducer.

Assuming the fraction of unit thickness of the sample occupied by the embedded material is $u$ then, the remainder, $1-u$, is the fraction of unit thickness of the composite occupied by matrix material. The transit time of signal in the medium can be written as;

$$t = t_1 + t_2 \qquad (2)$$

in which $t_1$ and $t_2$ are transit times of ultrasound signal in the embedded and matrix material, respectively. In a sample with the thickness of d, we have;

$$t_1 = \frac{ud}{c_1} \qquad (3)$$

$$t_2 = \frac{(1-u)d}{c_2} \qquad (4)$$

where $c_1$ and $c_2$ are the velocity of ultrasound in the embedded and matrix material, respectively. Substituting in equation 2, then:

$$t = \frac{ud}{c_1} + \frac{(1-u)d}{c_2} = d\left(\frac{u}{c_1} + \frac{1-u}{c_2}\right) \qquad (5)$$

using the simple equation;

$$c = \frac{d}{t}$$

and substituting in equation 5, we have:

$$\frac{1}{c} = \frac{u}{c_1} + \frac{(1-u)}{c_2} \qquad (6)$$

Assuming a homogenous distribution of particles in the matrix and the average number of particles in unit volume of the sample to be $N$, and if the particles in the sample are spherical with an average radius of $r$, then the value of $u$ in a unit distance is;

$$u = (Vem)^{1/3} = \left(N \times \frac{4}{3} \times \pi r^3\right)^{1/3} = 1.6rN^{1/3} \qquad (7)$$

$V_{em}$ is the fraction of unit volume occupied by embedded particles. For the general case (with embedded particles with arbitrary shape) instead of using equation 7, one can use equation $u = mN^{1/3}$ where $m$ is the shape factor to be determined. Combining equations 6 and 7 then;

$$c = \frac{1}{1.6N^{1/3} r\left(\frac{c_2 - c_1}{c_1 c_2}\right) + \frac{1}{c_2}} \qquad (8)$$

To calculate relative proportions of the two components, each mass is usually measured. To obtain $n$ (number of particles in the composite), the equation used is;

$$M_{em} = \rho V_{em} = \rho \times 4\ \pi r^3 n \qquad (9)$$

and therefore:

$$n = \frac{3M_{em}}{4\rho\pi r^3} \qquad (10)$$

where, $M_{em}$, $V_{em}$ and $\rho$ are the total mass, volume and density of the embedded material in the matrix, respectively. The value of $n$ obtained is total number of embedded particles in the total volume of the sample. But in equation 8 we need to know the number of particles per unit volume($N$), and it is therefore necessary to measure the total volume of the sample ($V$) and using the equation of $N = n/V$; to obtain $n$. Alternatively, $u$ (the fraction of the length occupied by embedded particles) required for equation 6 can be written as;

$$u = \left(\frac{V_{em}}{V}\right)^{1/3} = \left(\frac{M_{em}}{\rho V}\right)^{1/3} \qquad (11)$$

and therefore, it may be esaier to write equation 7 in terms of mass and density of the embedded particles and total volume of the sample, hence, we have;

$$c = \frac{1}{\left(\frac{M_{em}}{dV_{tot}}\right)^{1/3}\left(\frac{c_2 - c_1}{c_2 c_1}\right) + \frac{1}{c_2}} \qquad (12)$$

$$c = \frac{(\rho V)^{1/3} c_1 c_2}{M_{em}^{1/3}(c_2 - c_1) - (\rho V)^{1/3} c_1} \qquad (13)$$

Equation for the composite of glass beads in perspex with different porportions of glass beads is drawn in Figure 3.

Experimental results for two types of glass beads are also



[ Downloaded from mjiri.iums.ac.ir on 2026-11-22 ]

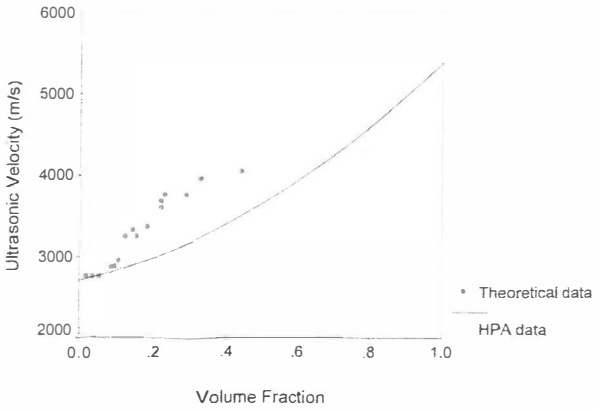



**Fig. 3.** Velocity of ultrasound in perspex-glass beads at different volume fractions.

shown in this figure.

The results for HAP- perspex are shown in Fig. 4, along with the theoretical curves using equation 6.

From the curves it is clear that the above calculation gives a slightly higher velocity compared with the experimental results. It may be concluded that there is a lag in transferring energy from perspex to glass beads at the interfaces which causes the velocity to be less than would be, regarding serial addition of the time taken for ultrasound to transmit through each of the components. It is clear that the experimental and theoretical results predicted by the above theory are very well matched for the case of perspex-glass beads, but there is less agreement between the theoretical and experimental results in the case of HAP-perspex composition.

The difference in behaviour between the samples of HAP-perspex and perspex-glass beads may arise from the fact that HAP and perspex, unlike perspex glass beads, are not inert in response to each other.

## REFERENCES

1. Kanis J, Melton LJ, Christiansene C, et al: The diagnosis of osteoporosis. J Bone Mineral Res 3: 1137-41, 1994.
2. Langton CM, Evans GP, Hodgekinson R, et al: Ultrasonic elastic and structural properties of cancellous bone, in current research in osteoporosis and bone mineral measurement. Conference on Osteoporosis and Bone Mineral Measurement. London: British Institute of Radiology, 1990.
3. Speller RD, Reyle GR, Horrocks JA: Instrumentation and techniques in bone density measurment. J Phys Sci Instrum 22: 202-14, 1989.
4. Tavakoli MB, Evans JA: The effect of bone structure on ultrasonic attenuation and velocity. Ultrasonic 30: 389-95, 1992.
5. Adams JE, Harrison EI, Alsop CW, et al: Ultrasound for bone densitometry: a comparison of three scanners. Osteoporosis Int 3: 55, 1998.
6. Sowers M, Lannausch M, Scholl T, et al: The reproducibility of ultrasound bone measure in triethnic population of pregnant adolescent and adult women. J Bone Mineral Res 13: 1768-74, 1988.
7. Kinra, VK, Patratis MS and Datta SK: Ultrasonic wave propagation in a random particular composite. Int J Solids Structures 16: 301, 1980.
8. Lakes R, Yoon HS, Katz J: Ultrasonic wave propagation and attenuation in wet bone. J Biomed Engin 8: 143-8, 1985.
9. Tavakoli MB, Evans JA: Ultrasonic velocity and attenuation in bone and bone elements. Phys Med Biol 36: 1529-37, 1991.
10. Hashin Z, Shtrikaman S: A variational approach to the theory of the elastic behaviour of multiphasic materials. J Mech Phys Solids 11: 127-40, 1963.
11. Katz JL: Hard tissue as composite material. J Biomech 4: 455-3, 1971.
12. Lees S, Davidson CL: The role of collagen in the elastic properties of calcified tissues. J Biomech 10: 475-86, 1977.
13. Less S, Davidson CL: Ultrasonic measurement of some mineral filled plastics; IEEE Trans Sonics Ultrasonic SU-24: 222-5, 1977.

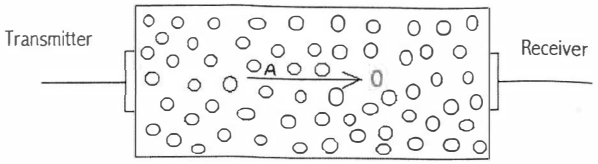



**Fig. 4.** Velocity of ultrasound in perspex-HAP at different volume fractions.



[ Downloaded from mjiri.iums.ac.ir on 2026-11-22 ]

[ Downloaded from mjiri.iums.ac.ir on 2026-11-22 ]